

APPROXIMATION OF HOLOMORPHIC FUNCTIONS IN BANACH SPACES

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Abstract. Let X be a complex Banach space. Recall that X admits a *finite dimensional Schauder decomposition* if there exists a sequence of finite dimensional subspaces of X , $\{X_n\}_{n=1}^{\infty}$, such that every $x \in X$ has a unique representation of the form $x = \sum_{n=1}^{\infty} x_n$, with $x_n \in X_n$ for every n . The finite dimensional Schauder decomposition is said to be *unconditional* if, for every $x \in X$, the series $x = \sum_{n=1}^{\infty} x_n$, which represents x , converges unconditionally, that is, $\sum_{n=1}^{\infty} x_{\pi(n)}$ converges for every permutation π of the integers. For short, we say that X admits an unconditional F.D.D.

We show that if X admits an unconditional F.D.D., then the following Runge approximation property holds

(R.A.P.) *There is $r \in (0, 1)$ such that for any $\epsilon > 0$, and any holomorphic function f on the open unit ball of X , there exists a holomorphic function h on X satisfying $|f - h| < \epsilon$ on the open ball of radius r centered at the origin.*